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Cosmic Ether

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A prerelativistic approach to particle dynamics is explored in an expanding Robertson–Walker cosmology. The receding galactic background provides a distinguished frame of reference and a unique cosmic time. In this context the relativistic, purely geometric space-time concept is criticized. Physical space is regarded as a permeable medium, the cosmic ether, which effects the world-lines of particles and rays. We study in detail a Robertson–Walker universe with linear expansion factor and negatively curved, open three-space; we choose the permeability tensor of the ether in such a way that the semiclassical approximation is exact. Galactic red-shifts depend on the refractive index of the ether. In the local Minkowskian limit the ether causes a time variation of mass, which scales inversely proportional to cosmic time. In the globally geodesic rest frames of galactic observers the ether manifests itself in an unbounded speed of signal transfer, in bifurcations of world-lines, and in time inversion effects.

1. INTRODUCTION

The Galilean and the special principle of relativity are associated with spaces that are isotropic, homogeneous, and essentially void. Minkowski space simply means the void, the vacuum, in which events are labeled by coordinate axes in space and time. In cosmology, however, space is generated by the galactic grid; this is the essence of cosmic space, and this grid provides a distinguished frame of reference. The uniform galactic recession is most simply described by a Robertson–Walker (RW) geometry with the line element $ds^2 = -c^2 d\tau^2 + a^2(\tau)\, d\sigma^2$ (comoving coordinates). Here $a^2(\tau)\, d\sigma^2$ denotes the line element of the cosmic 3-space, and the expansion factor $a(\tau)$ defines the length scale on this space at a given instant of cosmic time τ. Observers comoving with the galactic background see the galaxies isotropically receding in the constantly curved 3-space, whose curvature radius scales with the

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expansion factor. In the comoving frame all galaxies and galactic observers have constant space coordinates; their mutual recession is a mere consequence of the expansion of the 3-space, generated by the scale factor $a(\tau)$.

The comoving frame can be regarded as the universal frame of rest shared by all galactic observers; it determines an absolute space as well as an absolute cosmic time τ. Any coordinate transformation involving time will change the form of the RW line element, and then the time separation of infinitesimally neighboring events would no longer be the differential $d\tau$. Observers moving in this universal frame of rest will see an anisotropic galactic background. Accordingly, rest as defined above (by constant space coordinates in the comoving frame) and uniform motion are physically distinguishable states in this space-time conception based on the galactic background.

In this context it is very tempting to assume that physical space as generated by the expanding galactic grid is not a mere geometric construct, but has itself substance. This substance, the ether (Whittaker, 1951), manifests itself by its permeability. The coupling of light rays to the permeability tensor is effected as in a dielectric medium. In the eikonal equation the space-time metric is replaced by the permeability tensor. For particles we do the same in the Hamilton–Jacobi equation. With the usual appeal to cosmic homogeneity and isotropy, the permeability tensor takes the form $d\hat{s}^2 = -c^2\hat{h}^2(\tau)\, d\tau^2 + \hat{b}^2(\tau)\, d\sigma^2$, with two scale factors $\hat{h}(\tau)$ and $\hat{b}(\tau)$.

In Section 2 we study the dynamics of particles and rays in the cosmic ether. Energy and momentum are defined, and the galactic red-shift is discussed in comoving coordinates. We consider in this paper a RW cosmology with linear expansion factor and negatively curved 3-space. We choose the scale factors in the permeability tensor so that the semiclassical approximation is exact. This is not really necessary, but simplifies calculations, so that we need not resort to asymptotic reasoning. We derive the local Minkowskian vacuum limit, and in this limit a variation of mass in cosmic time.

In Section 3 we discuss world-lines in globally geodesic coordinates, by using the fact that the RW geometry is isometric to the forward light cone. In these coordinates the velocity of particles and rays in the ether is no longer bounded from above during the cosmic evolution. The world-lines may have inversion points, and they may even split. A single classical point particle in the comoving frame may appear in the geodesic rest frame of a galactic observer at different places at the same time!

In Section 4 we study red-shifts in globally geodesic rest frames. They appear in these coordinates as Doppler shifts depending on the refractive index of the ether. Since the semiclassical approximation is exact, we can derive the group and phase velocities of a wave field directly from the classical action; in a geodesic frame these velocities may have opposite orientation. Energy is proportional to frequency, but it is no longer positive in geodesic frames, and even not bounded from below. In the comoving frame energy is of course positive definite; cf. Section 2.

In Section 5 we investigate signal transfer in the ether. We consider two galactic observers, and study how the trajectory of the signal appears in their respective geodesic rest frames. The causality principle is preserved in the comoving galactic frame, namely, that every effect has a cause, that this cause precedes the effect, and that the decision on what is cause and effect is unambiguous and observer-independent (Tomaschitz, 1996). In the geodesic rest frames of galactic observers, however, causality may appear completely distorted due to time inversion and the mentioned splitting of the signal trajectories. In Section 6 we present our conclusions.

2. PERMEABILITY OF SPACE-TIME AND COSMOLOGICAL RED-SHIFT

We consider a RW cosmology with open 3-space and linear expansion factor $a(\tau) = cR^{-1}\tau$,

$$ds^2 = -c^2 d\tau^2 + a^2(\tau)\, d\sigma^2, \qquad d\sigma^2 = R^2 u^{-2}(d\xi_1^2 + d\xi_2^2 + du^2) \tag{2.1}$$

Cosmic time τ ranges in $[0, \infty]$, and $d\sigma^2$ is the line element of the Poincaré half-space H^3 (Ahlfors, 1981). (ξ_1, ξ_2, u), $u > 0$, are Cartesian half-space coordinates; H^3 has constant negative curvature $-1/R^2$. In the following we put $c = R = 1$.

This RW geometry is a flat 4-manifold, and can isometrically be mapped onto (the interior of) the forward light cone, $t^2 - |\mathbf{x}|^2 > 0$, $t > 0$ (Infeld and Schild, 1945). The coordinate change

$$\tau = \sqrt{t^2 - |\mathbf{x}|^2}, \qquad u = \frac{1}{t - x}\sqrt{t^2 - |\mathbf{x}|^2}$$
$$\xi_1 = \frac{y}{t - x} \qquad \xi_2 = \frac{z}{t - x} \tag{2.2}$$

transforms (2.1) into the Minkowski line element $ds^2 = -dt^2 + |d\mathbf{x}|^2$, $\mathbf{x} = (x, y, z)$. The inverse of (2.2) is

$$t = \frac{\tau}{2u}(\xi_1^2 + \xi_2^2 + u^2 + 1), \qquad x = \frac{\tau}{2u}(\xi_1^2 + \xi_2^2 + u^2 - 1)$$
$$y = \frac{\tau}{u}\xi_1, \qquad z = \frac{\tau}{u}\xi_2 \tag{2.3}$$

We assume the permeability tensor of space-time to be, like the metric, homogeneous and isotropic,

$$d\hat{s}^2 = -h^2(\tau)\, d\tau^2 + b^2(\tau)\tau^2 u^{-2}(d\xi_1^2 + d\xi_2^2 + du^2) \tag{2.4}$$

where $h(\tau)$ and $b(\tau)$ are functions of cosmic time τ. The proper orthochronous Lorentz group $SO^+(3,1)$ is the symmetry group of the spacelike sections $\tau =$ const of this line element (Ahlfors, 1981). The explicit group action leaving (2.4) invariant is (we use here complex notation, $\xi = \xi_1 + i\xi_2$)

$$\tau \to \tau, \qquad (\xi, u) \to (|c\xi + d|^2 + |c|^2 u^2)^{-1}[(a\xi + b)\overline{(c\xi + d)} + a\bar{c}u^2, u] \tag{2.5}$$

where $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ is an element of $SL(2, \mathbb{C})/\{\pm id\}$, which is isomorphic to $SO^+(3,1)$.

Throughout this paper we specify

$$h(\tau) = H/\tau, \qquad b(\tau) = B/\tau \tag{2.6}$$

Note that the permeability tensor is only conformal to the RW metric (2.1) if $H = B$ (arbitrary positive constants).

We consider particle trajectories along the u-semiaxis of H^3 (corresponding to the x-axis in the forward light cone), and so we may put in the preceding formulas $\xi_1 = \xi_2 = y = z = 0$. This is without loss of generality, since H^3 is a homogeneous space. We define the coupling of a point particle to the permeability tensor via the Hamilton–Jacobi equation,

$$\hat{g}^{-100} p_0 p_0 + \hat{g}^{-1uu} p_u p_u = -m^2 \varepsilon \tag{2.7}$$

with the generalized momenta $p_0 = \partial S/\partial\tau$ and $p_u = \partial S/\partial u$ substituted. The permeability tensor $\hat{g}_{\mu\nu}$ is defined by the line element (2.4) and (2.6) ($\hat{g}^{-100} = -H^{-2}\tau^2$, $\hat{g}^{-1uu} = B^{-2}u^2$), and ε may take the values ± 1 or 0. The action for particles moving along the u-semiaxis is easily calculated as

$$S = mB\nu \log u - mH(\nu^2 + \varepsilon)^{1/2} \log \tau \tag{2.8}$$

If $\varepsilon = -1$, then the integration parameter ν is restricted to $|\nu| > 1$, and if $\varepsilon = 0$, we assume $\nu \neq 0$. If $\varepsilon = 1$, there are no restrictions on ν. If we put $m = 1$ and $\varepsilon = 0$ in (2.8), S becomes the eikonal of geometric optics. If $\varepsilon = -1$, this means a negative mass square and superluminal motion (Bilaniuk *et al.*, 1962; Feinberg, 1967).

We obtain from the Lagrangian

$$L = -m\sqrt{|\hat{g}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu|} \tag{2.9}$$

$[x^\mu = (\tau, u)]$ the equations of motion

$$H^2(\dot{\tau}/\tau)^2 - B^2(\dot{u}/u)^2 = \varepsilon, \qquad \dot{u}/u = \text{const} \tag{2.10}$$

and the world-lines

$$\tau(s) = \exp(\sqrt{\nu^2 + \varepsilon}\, H^{-1} s), \qquad u(s) = \kappa \exp(\nu B^{-1} s) \tag{2.11}$$

The curve parameter s ranges over $\mathbb{R}$, κ is a positive integration constant, and ε and ν are the same constants as in (2.8). We have $\hat{g}^{-100}p_0 = m\dot{\tau}$, and $\hat{g}^{-1uu}p_u = m\dot{u}$. Eliminating the curve parameter s, the world-lines read

$$u = \kappa\tau^{\alpha}, \qquad \alpha := HB^{-1}\nu(\nu^2 + \varepsilon)^{-1/2} \tag{2.12}$$

which could have also been obtained from $\partial S/\partial\nu = \text{const}$. We consider here only trajectories along the u-semiaxis of H^3; all other trajectories can be obtained by applying the group action (2.5) onto (2.12).

We define energy and momentum as

$$E := p^0 = mH\sqrt{(\nu^2 + \varepsilon)}\tau^{-1}, \qquad p = p^u = mB\nu u\tau^{-2}, \qquad |p| = mB|\nu|\tau^{-1} \tag{2.13}$$

[Note that we raise and lower indices always with the space-time metric $g_{\mu\nu}$ as defined by the line element (2.1), e.g., $p^u = g^{uu}p_u$]. This definition of energy is also consistent with the semiclassical approximation, which is exact in this case; the classical action (eikonal) is identical with the phase of the spectral elementary waves of scalar, Dirac, and Maxwell equations coupled to the permeability tensor; the surfaces of constant action are the wavefronts. The coupling of wave equations to the permeability tensor of the ether is discussed in Tomaschitz (1998a). We obtain, with metric (2.1) and permeability tensor as defined in (2.4) and (2.6), the phase

$$\varphi = s\log u - H(B^{-2}s^2 + \varepsilon m^2)^{1/2}\log\tau \tag{2.14}$$

for plane waves propagating along the u-semiaxis of H^3. This phase is identical with the action (2.8) if we identify the spectral parameter s and the integration constant ν as $s = mB\nu$ (Tomaschitz, 1993). Since the semiclassical approximation is exact, it is almost compulsory to define energy in such a way that it is proportional to the frequency of the elementary waves, $E = \hbar\omega$ ($\hbar = 1$ in the following).

The group velocity is identical with the velocity of the classical point particle, $|\mathbf{v}| = \tau u^{-1}|du/d\tau|$, namely

$$|\mathbf{v}_{gr}| = |\mathbf{v}| = |\alpha| \tag{2.15}$$

with α as in (2.12). The phase velocity reads

$$|\mathbf{v}_{ph}| = |\beta|, \qquad \beta := HB^{-1}\nu^{-1}(\nu^2 + \varepsilon)^{1/2} \tag{2.16}$$

Because the classical action is identical with the phase (2.14), these velocities are obtained by equating the (τ, u)-differential of $\partial S/\partial\nu$, and S, respectively, to zero; see also (4.4). [The absolute values of the velocities and the momentum in (2.13) are of course taken with respect to the 3-space metric g_{ij} defined by the line element $a^2d\sigma^2$; cf. (2.1).] We have $|\mathbf{v}_{gr}||\mathbf{v}_{ph}| = H^2B^{-2}$. In comoving coordinates group and phase velocity always point into the same direction determined by the sign of the integration constant ν.

We may write the Hamilton–Jacobi equation (2.7) as

$$-\hat{c}^{-2}S_{,\tau}^{2} + u^{2}\hat{R}^{-2}S_{,u}^{2} = -\varepsilon\hat{m}^{2}\hat{c}^{2} \tag{2.17}$$

with the speed of light $\hat{c} = cHB^{-1}$, the curvature radius $\hat{R} = c\tau$ of the 3-space, and the time-dependent mass $\hat{m} = mB^{2}H^{-1}Rc^{-1}\tau^{-1}$. (We depart here from the convention $c = R = 1$.) If we also restore the natural units in (2.13), and express the integration parameter $|\nu|$ in terms of the velocity $|\mathbf{v}|$ via (2.15), then we obtain the familiar formulas

$$E = \frac{\hat{m}\hat{c}^{2}}{\sqrt{\varepsilon(1 - |\mathbf{v}|^{2}/\hat{c}^{2})}}, \qquad |p| = \frac{\hat{m}|\mathbf{v}|}{\sqrt{\varepsilon(1 - |\mathbf{v}|^{2}/\hat{c}^{2})}} \tag{2.18}$$

The energy of an observer comoving with the galactic background is not constant, nor is the energy of galaxies, $E_{gx} = cRmH\tau^{-1} = \hat{m}\hat{c}^{2}$ (integration parameters $\varepsilon = 1$ and $\nu = 0$).

From (2.17) we can easily obtain the local Minkowskian limit. On the time scale of a local physical process the mass of the particle and the curvature radius of the 3-space can be regarded as constant, i.e., at a given instant of cosmic time, $\hat{m} \approx \hat{m}(\tau_0)$, $\hat{R} \approx \hat{R}(\tau_0)$. Because H^3 is homogeneous, we may assume that the process takes place in a vicinity Δu of $u_0 = \hat{R}(\tau_0)$. If $\Delta u/\hat{R}(\tau_0) << 1$, we can put $u/\hat{R}(\tau_0) \approx 1$ in (2.17), and so obtain the local Minkowskian limit of the Hamilton–Jacobi equation. This limit is only approximately Minkowskian and unrelated to the geodesic frames introduced in the next section. In the cosmology discussed here [with scale factors as in (2.1) and (2.6)] mass scales inversely proportional to cosmic time, and the speed of light is constant. $\hat{m}(\tau_0)$ and $\hat{c}$ are the measured mass and speed of light, respectively, in a local Minkowskian frame; c and m are bare quantities.

Let us finally discuss the effect of the ether on the cosmological red-shift. For frequency and wavelength we have [cf. (2.13)]

$$\omega = E, \qquad \lambda\!\!\!^{-} = 1/|p| = (mB|\nu|)^{-1}\tau \tag{2.19}$$

Assume that an observer O_1 is placed at (space coordinate) $u_{abs} = 1$, and an observer O_2 at u_{em}, $0 < u_{em} < 1$. Observer O_2 emits a signal at time τ_{em} which is absorbed by O_1 at a later instant τ_{abs}. This signal [not necessarily a ray ($\varepsilon = 0$)] moves according to (2.12) with $\alpha > 0$, otherwise it would not reach O_1. (If $u_{em} > 1$, α must be negative.) Thus we have $\kappa = u_{em}\tau_{em}^{-\alpha}$, and $\tau_{abs} = \kappa^{-1/\alpha} = u_{em}^{-1/\alpha}\tau_{em}$. Since the wavelength is increasing with cosmic time τ, we obtain a red-shift, $\lambda\!\!\!^{-}(\tau_{abs})/\lambda\!\!\!^{-}(\tau_{em}) = u_{em}^{-1/\alpha} > 1$. If $\varepsilon = 0$, we have $n = 1/|\alpha| = B/H$ as refractive index of the ether [$\hat{c} = c/n$; cf. (2.17)]. The coordinate distance between the two observers is $\tau_{em}|\log u_{em}|$ and $\tau_{abs}|\log u_{em}|$ at emission and absorption time, respectively.

3. WORLD-LINES IN GLOBALLY GEODESIC COORDINATES

Applying the coordinate transformation (2.2) to the line element (2.4), we obtain for the permeability tensor $\hat{g}_{\mu\nu}$ in the forward light cone

$$\begin{aligned} \hat{g}_{00} &= -(h^2 t^2 - b^2|\mathbf{x}|^2)(t^2 - |\mathbf{x}|^2)^{-1} \\ \hat{g}_{ij} &= b^2\delta_{ij} - (h^2 - b^2)(t^2 - |\mathbf{x}|^2)^{-1} x_i x_j \\ \hat{g}_{0i} &= (h^2 - b^2)(t^2 - |\mathbf{x}|^2)^{-1} t x_i \end{aligned} \tag{3.1}$$

with $x^{\mu} = (t,\mathbf{x})$, $d\hat{s}^2 = \hat{g}_{\mu\nu}dx^{\mu}dx^{\nu}$ and $h = h(\sqrt{t^2 - |\mathbf{x}|^2})$, $b = b(\sqrt{t^2 - |\mathbf{x}|^2})$.

We may write, partially diagonalizing,

$$\begin{aligned} d\hat{s}^2 &= \hat{g}_{00}[dt - (h^2 - b^2)(h^2t^2 - b^2|\mathbf{x}|^2)^{-1} t x_i dx^i]^2 + \hat{\gamma}_{ij} dx^i\, dx^j \\ \hat{\gamma}_{ij} &= b^2[\delta_{ij} - (h^2 - b^2)(h^2t^2 - b^2|\mathbf{x}|^2)^{-1} x_i x_j] \end{aligned} \tag{3.2}$$

By rotating x_i into the x-axis we see that $\hat{\gamma}_{ij}$ is positive definite only if $\hat{g}_{00}$ is negative, but since γ_{ij} is not meant as a metric on the 3-space, this is not required. In the same way we find the determinants $\hat{\gamma} = h^2 b^6 (h^2t^2 - b^2|\mathbf{x}|^2)^{-1}(t^2 - |\mathbf{x}|^2)$, and $\hat{g} = \hat{g}_{00}\hat{\gamma} = -b^6h^2$. The inverse of γ_{ij} reads

$$\hat{\gamma}^{-1ij} = b^{-2}[\delta^{ij} + h^{-2}(h^2 - b^2)(t^2 - |\mathbf{x}|^2)^{-1} x^i x^j] \tag{3.3}$$

and for the inverse of $\hat{g}_{\mu\nu}$ we obtain

$$\begin{aligned} \hat{g}^{-100} &= \frac{t^2|\mathbf{x}|^2(h^2 - b^2)^2 - h^2b^2(t^2 - |\mathbf{x}|^2)^2}{h^2b^2(h^2t^2 - b^2|\mathbf{x}|^2)(t^2 - |\mathbf{x}|^2)} \\ \hat{g}^{-1ij} &= \hat{\gamma}^{-1ij}, \\ \hat{g}^{-1i0} &= h^{-2}b^{-2}(h^2 - b^2)(t^2 - |\mathbf{x}|^2)^{-1} t x^i \end{aligned} \tag{3.4}$$

The line element of the permeability tensor (3.1) is Lorentz-invariant, because the Lorentz group in its standard representation in Minkowski space corresponds via the coordinate transformation (2.2), (2.3) to the group action (2.5), which leaves (2.4) invariant. This invariance can also be checked directly, since $d\hat{s}^2$ as defined after (3.1) is evidently invariant under orthogonal rotations, and it is easy to verify that this line element is also invariant under a Lorentz boost along the x-axis.

Inserting (2.11) into (2.3), we obtain for the world-lines in the forward light cone

$$\begin{pmatrix} t(s) \\ x(s) \end{pmatrix} = \exp(\sqrt{\nu^2 + \varepsilon}H^{-1}s)\begin{pmatrix} \cosh(\nu B^{-1}s + \log\kappa) \\ \sinh(\nu B^{-1}s + \log\kappa) \end{pmatrix} \tag{3.5}$$

The same restrictions apply to the parameters ε, ν, and κ as in (2.8) and (2.11). From (3.5) we have

$$x/t = \tanh(\nu B^{-1}s + \log \kappa), \qquad s = H(\nu^2 + \varepsilon)^{-1/2} \log\sqrt{t^2 - x^2} \quad (3.6)$$

$$dt/ds = (\nu^2 + \varepsilon)^{1/2} H^{-1} t(1 + \alpha x/t)$$
$$dx/ds = (\nu^2 + \varepsilon)^{1/2} H^{-1} t(x/t + \alpha) \quad (3.7)$$

with $\alpha(\varepsilon, \nu) = HB^{-1}\nu(\nu^2 + \varepsilon)^{-1/2}$ as in (2.12). The velocity reads

$$\mathbf{v} = dx/dt = (x/t + \alpha)(1 + \alpha x/t)^{-1} \quad (3.8)$$

By eliminating the curve parameter s, we obtain for the world-lines in these nonseparable coordinates

$$1 = \kappa(t - x)^{(\alpha + 1)/2}(t + x)^{(\alpha - 1)/2} \quad (3.9)$$

The integration constants κ and α relate to the initial values (t_0, x_0) (in the forward light cone) and $\mathbf{v}_0$ (initial velocity) as follows. From (3.9) we determine

$$\kappa(t_0, x_0, \alpha) = (t_0 - x_0)^{-(1+\alpha)/2}(t_0 + x_0)^{(1-\alpha)/2} \quad (3.10)$$

and from (3.8) we obtain $\alpha(x_0/t_0, \mathbf{v}_0)$. An initial velocity of $\mathbf{v}_0 = t_0/x_0$ is not permitted, since that would need $\varepsilon = -1$, $\nu = \pm 1$. But otherwise there are no restrictions on the size of $\mathbf{v}_0$.

The qualitative behavior of the trajectories (3.5), (3.9) is determined by the parameter α:

(I) $|\alpha| < 1$. The velocity (3.8) can never diverge, since $|x/t| < 1$ in the forward light cone. However, $\mathbf{v}$ can become zero; the trajectory has a turning point at (t_{inv}, x_{inv}) given by

$$s_{inv} = B\nu^{-1} \log[\kappa^{-1}(1 - \alpha)^{1/2}(1 + \alpha)^{-1/2}]$$
$$t_{inv} := t(s_{inv}) = (1 - \alpha^2)^{-1/2}[\kappa^{-1}(1 - \alpha)^{1/2}(1 + \alpha)^{-1/2}]^{1/\alpha} \quad (3.11)$$
$$x_{inv} := x(s_{inv}) = -\alpha t_{inv}$$

s_{inv} is defined by $dx/ds = 0$, which requires $x/t + \alpha = 0$; cf. (3.7) and (3.8). A short inspection of the trajectories (3.5) gives: If $s \to \infty$, then $t \to \infty$ and $x \to \mathrm{sign}(\alpha)\infty$. If $s \to -\infty$, then $t \to 0$ and $x \to 0$.

(a) If $\alpha > 0$, the trajectory starts at $(t, x) = (0, 0)$, then the particle moves along the negative x-axis to (t_{inv}, x_{inv}), $x_{inv} < 0$. Here a velocity inversion occurs, and it moves back to $x = 0$ and onward to $x = \infty$, which is reached at $t = \infty$.

(b) If $\alpha < 0$, the particle moves at first along the positive x-axis until it reaches x_{inv} ($x_{inv} > 0$), and then with negative velocity to $x = -\infty$. Otherwise as in (a).

For future reference we mention here that in either case the particle reaches the coordinate origin $x = 0$ at time

$$t_o := t(s_o) = \kappa^{-1/\alpha}, \qquad s_o = -B\nu^{-1} \log \kappa \tag{3.12}$$

Clearly, $s_{inv} < s_o$.

(II) $|\alpha| > 1$. The velocity cannot become zero, but it may diverge. It becomes singular at

$$\begin{aligned} s_\infty &= B\nu^{-1} \log[\kappa^{-1}(\alpha - 1)^{1/2} (\alpha + 1)^{-1/2}] \\ t_\infty &:= t(s_\infty) = |\alpha|(\alpha^2 - 1)^{-1/2}[\kappa^{-1}(\alpha - 1)^{1/2}(\alpha + 1)^{-1/2}]^{1/\alpha} \\ x_\infty &:= x(s_\infty) = -\alpha^{-1}\, t_\infty \end{aligned} \tag{3.13}$$

s_∞ is defined by $dt/ds = 0$, which requires $1 + \alpha x/t = 0$. $t(s)$ attains its minimum value at s_∞. The asymptotic behavior of the trajectory is as follows. If $s \to \infty$, then $t \to \infty$ and $x \to \mathrm{sign}(\alpha)\infty$. If $s \to -\infty$, then $t \to \infty$ and $x \to -\mathrm{sign}(\alpha)\infty$. So there are actually two different trajectories, corresponding to two different particles P_1 and P_2. The parameter range of trajectory P_1 is $[s_\infty, \infty]$, and P_2 is defined by the range $[-\infty, s_\infty]$. Clearly, P_1 and P_2 correspond to one and the same particle in the comoving frame. This bifurcation is similar to the double images of tachyons pointed out in Tomaschitz (1997, 1998b).

(a) $\alpha > 0$: Particle P_1 starts at (t_∞, x_∞), $x_\infty < 0$, and moves on, with positive velocity, to $x = \infty$, which is reached at $t = \infty$. Particle P_2 likewise starts at (t_∞, x_∞), but moves with negative velocity in the opposite direction, and reaches $x = -\infty$ at $t = \infty$.

(b) $\alpha < 0$: Also here both particles start at (t_∞, x_∞), $x_\infty > 0$. Particle P_2 moves with positive velocity to $x = \infty$, and P_1 heads in the opposite direction to $x = -\infty$.

In both cases P_1 reaches the coordinate origin at parameter value s_o and time t_o; cf. (3.12).

(III) $|\alpha| = 1$. In this case the velocity (3.8) has neither zeros nor singularities. The asymptotics of (3.5) is as follows. If $s \to \infty$, then $t \to \infty$ and $x \to \mathrm{sign}(\alpha)\infty$. For $s \to -\infty$ we have, if $\alpha = 1$, $t \to 1/(2\kappa)$ and $x \to -1/(2\kappa)$. If $\alpha = -1$, we obtain in this limit $t = \kappa/2$ and $x = \kappa/2$.

(a) If $\alpha = 1$, the trajectory (3.9) reads $x = t - 1/\kappa$. It starts at $(t, x) = (1/(2\kappa), -1/(2\kappa))$, reaches $x = 0$ at t_o [cf. (3.12)] and moves to $x = \infty$.

(b) If $\alpha = -1$, we have $x = -t + \kappa$. The particle starts at $(\kappa/2, \kappa/2)$, reaches the coordinate origin at t_o, and moves on to $x = -\infty$.

4. RED-SHIFTS IN THE FORWARD LIGHT CONE

In globally geodesic coordinates we obtain, analogous to (2.8), for the action of a particle moving along the x-axis

$$S = mB\nu \log(t + x)^{1/2} (t - x)^{-1/2}$$
$$- mH(\nu^2 + \varepsilon)^{1/2} \log(t^2 - x^2)^{1/2} \quad (4.1)$$

which solves the Hamilton–Jacobi equation

$$\hat{g}^{-1tt} p_0 p_0 + \hat{g}^{-1xx} p_x p_x = -m^2\varepsilon \quad (4.2)$$

($p_0 = \partial S/\partial t$, $p_x = \partial S/\partial x$) with the inverse permeability tensor $\hat{g}^{-1\mu\nu}$ as defined in (3.4). Energy and 3-momentum read

$$E = \eta^{00}\partial S/\partial t = mB\nu(t^2 - x^2)^{-1} t(x/t + \beta)$$
$$p^x = \eta^{xx}\partial S/\partial x = mB\nu(t^2 - x^2)^{-1} t(1 + \beta x/t) \quad (4.3)$$

with $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$, and $\beta(\varepsilon, \nu) = HB^{-1}\nu^{-1}\sqrt{\nu^2 + \varepsilon}$ as in (2.16). Equations (4.1)–(4.3) may be obtained from (2.8), (2.7), and (2.13), respectively, via the coordinate change (2.2) and (2.3). For a galactic observer ($\nu = 0$, $\varepsilon = 1$) we have $E_{gx} = mHt(t^2 - x^2)^{-1}$ and $p^x_{gx} = -mHx(t^2 - x^2)^{-1}$. The phase velocity reads

$$\mathbf{v}_{ph} = -(\partial S/\partial t)(\partial S/\partial x)^{-1} = (x/t + \beta)(1 + \beta x/t)^{-1} \quad (4.4)$$

The group velocity $\mathbf{v}_{gr} = -(\partial^2 S/\partial\nu\partial t)(\partial^2 S/\partial\nu\partial x)^{-1}$ coincides with the classical particle velocity $\mathbf{v}$ as given in (3.8), since the semiclassical approximation is exact; cf. Section 2. By eliminating x/t from (3.8) and (4.4), we obtain

$$\mathbf{v}_{gr} = (\mathbf{v}_{ph} + \gamma)(1 + \gamma\mathbf{v}_{ph})^{-1}, \qquad \gamma := (\alpha - \beta)(1 - \alpha\beta)^{-1} \quad (4.5)$$

If $\alpha\beta = 1$, i. e., $H = B$, we have instead $\mathbf{v}_{gr}\mathbf{v}_{ph} = 1$. If $\varepsilon = 0$ (light), we have $\mathbf{v}_{gr} = \mathbf{v}_{ph}$. A zero of the phase velocity ($\mathbf{v}_{gr} = \gamma$) also means a sign change of the energy as discussed at the end of this section. The phase and the group velocity do not necessarily have the same sign. The ranges of $\mathbf{v}_{gr}$ and $\mathbf{v}_{ph}$ in which these velocities are opposite can easily be read off from the hyperbola (4.5).

Frequency and wavelength read now, analogous to (2.18),

$$\omega = E, \qquad \lambda\!\!\!^{-} = 1/|p^x| = (t^2 - x^2)|mB\nu t(1 + \beta x/t)|^{-1} \quad (4.6)$$

so that $|\omega|\lambda\!\!\!^{-} = |\mathbf{v}_{ph}|$. The phase velocity $\mathbf{v}_{ph}$ is not constant, not even for light ($m = 1$, $\varepsilon = 0$). Note that ω may be negative; $|\omega|/(2\pi)$ is the number of vibrations in the ether per time unit.

Let us assume an observer O_2 who is placed on a galaxy that moves with constant velocity $\mathbf{v}$ along the x-axis. His rest frame (t', x') is related to the (t, x)-frame (rest frame of an observer O_1) by the Lorentz boost

$$t = (1 - \mathbf{v}^2)^{-1/2}(t' + \mathbf{v}x'), \qquad x = (1 - \mathbf{v}^2)^{-1/2}(x' + \mathbf{v}t') \quad (4.7)$$

The action (4.1) is Lorentz invariant, up to an irrelevant constant,

$$S(t, x) = S(t', x') + mB\nu \log(1 + \mathbf{v})^{1/2}(1 - \mathbf{v})^{-1/2} \tag{4.8}$$

Because of this invariance we obtain in the primed coordinate frame for energy E' momentum $p^{x'}$, frequency ω', wavelength λ', and group and phase velocities $\mathbf{v}'_{gr}$ and $\mathbf{v}'_{ph}$ the same formulas (4.3), (4.6), (3.8), and (4.4), respectively, with (t, x) replaced by (t', x').

Observer O_2 emits a signal at (t_0, x_0) (coordinates in the rest frame of O_1). Since O_2 is sitting on a galaxy, his velocity in the rest frame of O_1 is $\mathbf{v} = x_0/t_0$. The wavelength $\lambda_{em}(t_0, x_0, \nu, \varepsilon)$ of this signal in the (t, x)-frame is as in (4.6). The wavelength that observer O_2 actually emits and measures in his frame of rest (t', x') is

$$\lambda'_{em} = (t_0'^2 - x_0'^2)|mB\nu t'_0(1 + \beta x'_0/t'_0)|^{-1} = (mB|\nu|)^{-1} t_0 \sqrt{1 - x_0^2 / t_0^2} \tag{4.9}$$

To derive the second equation we used

$$t'\left(1 + \beta \frac{x'}{t'}\right) = (1 - \beta\mathbf{v})(1 - \mathbf{v}^2)^{-1/2}\, t\left(1 + \frac{\beta - \mathbf{v}}{1 - \beta\mathbf{v}} \frac{x}{t}\right) \tag{4.10}$$

and (t'_0, x'_0) is related to (t_0, x_0) via (4.7), of course.

In order to calculate the red-shift, we still have to know the wavelength of the signal [emitted at (t_0, x_0) with wavelength $\lambda_{em}(t_0, x_0, \nu, \varepsilon)$] when it arrives at $x = 0$, where observer O_1 is placed. With the initial values $(t_0, x_0, \varepsilon, \nu)$ we determine the integration parameter κ in (3.10). The signal arrives at $x = 0$ at time $t_{abs} = \kappa^{-1/\alpha}$; cf. (3.12). Therefore, according to (4.6), we have for the wavelength absorbed by O_1

$$\lambda_{abs} = \frac{t_{abs}}{mB|\nu|} = \frac{1}{mB|\nu|}\, t_0\sqrt{1 - x_0^2/t_0^2}\left(\frac{1 + x_0/t_0}{1 - x_0/t_0}\right)^{-1/(2\alpha)} \tag{4.11}$$

From (4.11) and (4.9) we obtain

$$\frac{\lambda_{abs}}{\lambda'_{em}} = \left(\frac{1 + x_0/t_0}{1 - x_0/t_0}\right)^{-1/(2\alpha)} \tag{4.12}$$

To summarize the notation in this formula: λ'_{em} is the wavelength of the signal measured in the rest frame (t', x') of observer O_2 at the time of its emission (by O_2). O_2 is placed on a galaxy with coordinates (t_0, x_0) and speed $\mathbf{v} = x_0/t_0$ in the rest frame of O_1. λ_{abs} is the wavelength observed and absorbed by O_1 at $x = 0$. Note that x_0 and α must have opposite signs, otherwise the signal emitted at x_0 cannot reach O_1; cf. the discussion at the end of Section 3, (I)–(III); so we safely have $\lambda_{abs} > \lambda'_{em}$. We may write (4.12) as the Doppler shift

$$\frac{\lambda_{abs}}{\lambda'_{em}} = \left(\frac{1+|\mathbf{v}|}{1-|\mathbf{v}|}\right)^{1/(2|\alpha|)}, \qquad |\alpha|^{-1} = BH^{-1}|\nu|^{-1}\sqrt{\nu^2+\varepsilon} \tag{4.13}$$

If $H = B$ and $\varepsilon = 0$, we obtain the vacuum Doppler shift, because under these conditions the permeability tensor is conformal to the Minkowski metric [cf. (3.1) and (2.6)] and the conformal factor drops out in the eikonal equation. Otherwise the red-shift depends on B/H, the refractive index in comoving coordinates; cf. the end of Section 2.

By the same reasoning as in (4.9) we obtain for the frequency of the emitted signal in the rest frame (t', x') of O_2,

$$\omega'_{em} = mB\nu(t_0'^2 - x_0'^2)^{-1}t_0'(\beta + x_0'/t_0') = mB\nu\beta t_0^{-1}(1 - x_0^2/t_0^2)^{-1/2} \tag{4.14}$$

The frequency absorbed by O_1 at $(t_{abs}, 0)$ is

$$\omega_{abs} = mB\nu\beta\kappa^{1/\alpha} = \frac{mB\nu\beta}{t_0\sqrt{1 - x_0^2/t_0^2}}\left(\frac{1 + x_0/t_0}{1 - x_0/t_0}\right)^{1/(2\alpha)} \tag{4.15}$$

which corresponds to (4.11). Thus we obtain

$$\frac{\omega_{abs}}{\omega'_{em}} = \left(\frac{1 + x_0/t_0}{1 - x_0/t_0}\right)^{1/(2\alpha)} \tag{4.16}$$

Although the phase velocity is not constant, we still have $\omega_{abs}/\omega'_{em} = \lambda'_{em}/\lambda_{abs}$, as in comoving coordinates; cf. the end of Section 2.

Remark. In Section 2 we need not distinguish between primed and unprimed quantities, since we use one and the same rest frame (the comoving frame) for all galaxies. In this section, however, we attach to every galactic observer an individual rest frame. These globally geodesic frames are related by Lorentz transformations.

Let us finally study the qualitative behavior of the energy in globally geodesic frames. For $|\beta| \geq 1$ the energy (4.3) is positive, since the relevant term $\nu(x/t + \beta)$ is. For $|\beta| < 1$, however, there will always be a sign change during the evolution, accompanied by a sign change of the phase velocity (4.4). With (3.5) we may write (4.3) as

$$E(s) = mB\nu \exp(-\sqrt{\nu^2 + \varepsilon}\, H^{-1}s) \times [\beta \cosh(\nu B^{-1}s + \log\kappa) + \sinh(\nu B^{-1}s + \log\kappa)] \tag{4.17}$$

The parameter value s_E and the coordinates at which $E(s)$ is zero read

$$s_E = B\nu^{-1}\log[\kappa^{-1}(1-\beta)^{1/2}(1+\beta)^{-1/2}]$$
$$t_E = (1-\beta^2)^{-1/2}[\kappa^{-1}(1-\beta)^{1/2}(1+\beta)^{-1/2}]^{1/\alpha}, \qquad x_E = -\beta t_E \quad (4.18)$$

This is quite similar to the inversion point of $\mathbf{v}_{gr}$; cf. (3.11). We always have $s_o > s_E$, i.e., the sign change happens before the particle reaches the origin; cf. (3.12). By comparing s_E to s_{inv} in (3.11) and s_∞ in (3.13), we can immediately derive the following statements. If $0 < \alpha < 1$ and $\alpha < \beta$ $(\alpha > \beta)$, the sign change of the energy happens prior to (after) the velocity inversion. If $-1 < \alpha < 0$ and $\alpha < \beta$ $(\alpha > \beta)$, the sign change happens after (prior to) the velocity inversion. If $\alpha > 1$ and $1/\alpha > \beta$ $(1/\alpha < \beta)$, the energy of particle P_1 (P_2) changes its sign. If $\alpha < -1$ and $1/\alpha > \beta$ $(1/\alpha < \beta)$, the energy of particle P_2 (P_1) changes its sign.

The energy is nonsingular. $E(s \to \infty)$ converges to $+\infty$ if $|\alpha| > 1$ and to $+0$ for $|\alpha| < 1$. If $|\alpha| = 1$, it attains a finite, positive limit $\frac{1}{2} mB|\nu|(1 + |\beta|)$. $E(s \to -\infty)$ converges to $+\infty$ if $|\beta| > 1$, and to $-\infty$ if $|\beta| < 1$. If $|\beta| = 1$, $E(s \to -\infty)$ converges to $+0$ if $H/B > 1$, to $mB|\nu|$ if $H/B = 1$, and to $+\infty$ if $H/B < 1$. The energy reaches an extremal value, $dE(s)/ds = 0$, at $x/t = \gamma$ [cf. (4.5)] provided $|\gamma| < 1$. We then have $\mathbf{v}_{ph} = \alpha$.

5. SIGNAL TRANSFER THROUGH THE ETHER

We study here a communication process between two galactic observers by means of signals freely traveling through the ether as defined in Sections 2 and 3. The coordinates (t,x) label the globally geodesic rest frame (forward light cone) of an observer O_1. In this frame the galactic world-lines [cf. (3.5) with $\nu = 0$, $\varepsilon = 1$] read

$$\begin{pmatrix} t(s) \\ x(s) \end{pmatrix} = \frac{1}{2} e^{s/H}(\kappa_{gx} \pm \kappa_{gx}^{-1}) \quad (5.1)$$

$$x = \mathbf{v}_{gx} t, \qquad \mathbf{v}_{gx} := (\kappa_{gx}^2 - 1)(\kappa_{gx}^2 + 1)^{-1} \quad (5.2)$$

The rest frame (t', x') of galaxy κ_{gx} is obtained by means of the Lorentz boost (4.7) (with $\mathbf{v} = \mathbf{v}_{gx}$). In the following we consider a galaxy moving along the positive x-axis so that $\mathbf{v}_{gx} > 0$ (i.e., $\kappa_{gx} > 1$). A signal (3.5) is emitted at $x = 0$; we denote its integration parameters by $\alpha(\varepsilon, \nu)$ [cf. (3.7)] and κ. This signal reaches the galaxy (at $x' = 0$) only if $\alpha > 0$; cf. the discussion after (3.10). It is absorbed there by an observer O_2 at

$$\begin{pmatrix} t_{abs} \\ x_{abs} \end{pmatrix} = \frac{1}{2}(\kappa_{gx}/\kappa)^{1/\alpha}(\kappa_{gx} \pm \kappa_{gx}^{-1}) \tag{5.3}$$

The corresponding values of the curve parameters of galaxy (5.1) and signal (3.5) are

$$s_{gx} = H\alpha^{-1}\log(\kappa_{gx}/\kappa), \qquad s_{abs} = B\nu^{-1}\log(\kappa_{gx}/\kappa) \tag{5.4}$$

The emission of the signal happens at [cf. (3.12)]

$$s_{em} = -B\nu^{-1}\log\kappa \tag{5.5}$$

$$t_{em} = \kappa^{-1/\alpha}, \qquad x_{em} = 0$$

[$t_{em} := t(s_{em})$ with $t(s)$ as in (3.5)]. We always have $0 < t_{em} < t_{abs}$.

If we apply the Lorentz boost (4.7) [$\mathbf{v} = \mathbf{v}_{gx}$ as in (5.2)] to the trajectory (3.5), it retains its shape in the primed coordinates apart from a rescaling of κ, $\kappa \to \kappa/\kappa_{gx}$,

$$\begin{pmatrix} t'(s) \\ x'(s) \end{pmatrix} = \exp(\sqrt{\nu^2 + \varepsilon}\, H^{-1}s)\begin{pmatrix} \cosh(\nu B^{-1}s + \log(\kappa/\kappa_{gx})) \\ \sinh(\nu B^{-1}s + \log(\kappa/\kappa_{gx})) \end{pmatrix} \tag{5.6}$$

The world-line (5.1) of galaxy κ_{gx} (observer O_2) reads in primed coordinates $t' = \exp(s/H), x' = 0$; the trajectory of observer O_1 [$t = \exp(s/H), x = 0$] reads

$$\begin{pmatrix} t'(s) \\ x'(s) \end{pmatrix} = \frac{1}{2}e^{s/H}(\kappa_{gx}^{-1} \pm \kappa_{gx}) \tag{5.7}$$

The parameter values at which the signal is emitted [namely s_{em} for the signal, cf. (5.5), and $s_{O_1} = -H\alpha^{-1}\log\kappa$ for the trajectory of observer O_1], as well as the parameter values at which the signal is absorbed [s_{gx} for the galaxy and s_{abs} for the signal; cf. (5.4)], remain the same in the primed frame. However, the parameter values (3.11) and (3.13) at which the velocity inversion and the splitting of the trajectory occurs do change in the primed frame,

$$s'_{inv} = B\nu^{-1}\log[(\kappa_{gx}/\kappa)(1-\alpha)^{1/2}(1+\alpha)^{-1/2}] \tag{5.8}$$

$$s'_{\infty} = B\nu^{-1}\log[(\kappa_{gx}/\kappa)(\alpha-1)^{1/2}(\alpha+1)^{-1/2}] \tag{5.9}$$

because of the rescaling of κ. In the rest frame (t', x') of galaxy κ_{gx} we obtain, by inserting the parameter values s_{em} and s_{abs} into (5.6),

$$\begin{pmatrix} t'_{em} \\ x'_{em} \end{pmatrix} = \frac{1}{2}\kappa^{-1/\alpha}(\kappa_{gx}^{-1} \pm \kappa_{gx}) \tag{5.10}$$

$$t'_{abs} = (\kappa_{gx}/\kappa)^{1/\alpha}, \qquad x'_{abs} = 0 \tag{5.11}$$

for emission and absorption events. We have $t'_{abs} > t'_{em}$, provided

$$|\alpha| < |\log \kappa_{gx}|[\log \tfrac{1}{2}(\kappa_{gx} + \kappa_{gx}^{-1})]^{-1} \tag{5.12}$$

If $|\alpha| < 1$, this inequality is always satisfied. Throughout this section we assume, as mentioned after (5.2), that $\kappa_{gx} > 1$ and $\nu > 0$ (i.e., $\alpha > 0$), which means that both galaxy and signal move along the positive x-axis: [For $\kappa_{gx} < 1$ and $\nu < 0$ equations (5.1)–(5.12) likewise hold; signal and galaxy then move in opposite directions.]

After these preparations we can easily figure out the qualitative behavior of the signal in the primed coordinate frame, namely, how the signal transfer appears to observer O_2 who is placed at galaxy κ_{gx} (at $x' = 0$). In the following $\kappa_{gx} > 1$ and $\alpha > 0$.

(I) $\alpha < 1$; cf. the discussion of (3.11). In this case $s_{abs} > s'_{inv}$ always holds, cf. (5.4) and (5.7).

(a) If $\alpha > (\kappa_{gx}^2 - 1)(\kappa_{gx}^2 + 1)^{-1}$ [i.e., $\kappa_{gx}(1 - \alpha)^{1/2}(1 + \alpha)^{-1/2} < 1$], then we also have $s_{em} > s'_{inv}$; cf. (5.5). So both emission and absorption happen after the velocity inversion (which actually means that no velocity inversion takes place). Observer O_2 sees the signal moving from $x'_{em} < 0$ straight onto him at $x'_{abs} = 0$.

(b) If $\alpha < (\kappa_{gx}^2 - 1)(\kappa_{gx}^2 + 1)^{-1}$, then $s_{em} < s'_{inv}$. Thus O_2 sees the signal again emitted at $x'_{em} < 0$, but moving at first away from him along the negative x-axis. Then the velocity inversion occurs, and the signal finally arrives with positive velocity at $x'_{abs} = 0$. In both cases (a) and (b) the emission happens prior to the absorption, $t'_{abs} > t'_{em}$; cf. (5.10) and (5.11).

(II) $\alpha > 1$; see the discussion of (3.13). We always have $s_{abs} > s'_{\infty}$; cf. (5.4) and (5.9).

(a) If $\alpha < (\kappa_{gx}^2 + 1)(\kappa_{gx}^2 - 1)^{-1}$ [i.e., $\kappa_{gx}(\alpha - 1)^{1/2}(\alpha + 1)^{-1/2} < 1$], then $s_{em} > s'_{\infty}$; cf. (5.5). Observer O_2 sees the particle P_1 [as defined after (3.13)] as first emitted at $x'_{em} < 0$, and then absorbed by him at $x'_{abs} = 0$. Particle P_1 moves straight to the observer with positive velocity. Also in this case we have $t'_{abs} > t'_{em}$.

(b) If

$$\alpha > (\kappa_{gx}^2 + 1)(\kappa_{gx}^2 - 1)^{-1} \tag{5.13}$$

then the trajectory of the classical point particle splits in the rest frame of O_2. We have $s_{em} < s'_{\infty} < s_{abs}$. Observer O_2 sees a particle P_1 emitted at $(t'_{\infty}, x'_{\infty})$, $x'_{\infty} < 0$, which moves toward him with positive velocity, and is absorbed by him at $x'_{abs} = 0$ [$(t'_{\infty}, x'_{\infty})$ is defined in (3.13) with κ replaced by κ/κ_{gx}]. Simultaneously he sees a second particle P_2, likewise emitted at $(t'_{\infty}, x'_{\infty})$; cf. the discussion after (3.13). This particle moves with negative velocity in the opposite direction and reaches x'_{em} ($x'_{em} < x'_{\infty} < 0$) at t'_{em}; cf. (5.10). We always have $t'_{\infty} < t'_{em}$ and $t'_{\infty} < t'_{abs}$. If $t'_{abs} < t'_{em}$, then O_2 absorbs particle P_1 prior to the arrival of P_2 at O_1. It is of course also possible that

$t'_{abs} = t'_{em}$. Even if $t'_{abs} > t'_{em}$, the observation of O_2 is qualitatively completely different from that what O_1 observes in his rest frame, namely a single particle moving straight from O_1 to O_2.

Remark. We have, for $\kappa_{gx} > 1$,

$$\log \kappa_{gx}[\log \tfrac{1}{2}(\kappa_{gx} + \kappa_{gx}^{-1})]^{-1} - (\kappa_{gx}^2 + 1)(\kappa_{gx}^2 - 1)^{-1} > 0 \qquad (5.14)$$

Therefore it is always possible to find an $\alpha > 0$ which satisfies both inequalities (5.12) and (5.13), so that $t'_{abs} > t'_{em}$. Inequality (5.14) can easily be proven by introducing $(\kappa_{gx}^2 + 1)(\kappa_{gx}^2 - 1)^{-1}$ as new variable and by differentiation. In the limit $\kappa_{gx} \to 1$ this inequality gets sharp, and we will most likely encounter a time inversion, $t'_{abs} < t'_{em}$.

(III) $\alpha = 1$; cf. the end of Section 3. There is no turning point, nor a splitting of the trajectory, nor a time inversion, and observer O_2 sees the signal coming straight onto him. His observation qualitatively agrees with that of O_1.

Let us finally discuss the energy transfer. In the rest frame (t, x) of observer O_1 the particle energy $E(s)$, parametrized by the curve parameter, is given in (4.17). In the rest frame (t', x') of O_2 we have for $E'(s)$ the same expression with κ replaced by κ/κ_{gx}. For emission and absorption energies in the respective frames we obtain with (5.4) and (5.5)

$$\begin{aligned}
E_{em} &= E(s_{em}) = mB\nu\beta\kappa^{1/\alpha} \\
E_{abs} &= E(s_{abs}) = \tfrac{1}{2} mB\nu(\kappa/\kappa_{gx})^{1/\alpha}[\beta(\kappa_{gx} + \kappa_{gx}^{-1}) + (\kappa_{gx} - \kappa_{gx}^{-1})] \qquad (5.15) \\
E'_{em} &= E'(s_{em}) = \tfrac{1}{2} mB\nu\kappa^{1/\alpha}[\beta(\kappa_{gx}^{-1} + \kappa_{gx}) + (\kappa_{gx}^{-1} - \kappa_{gx})] \\
E'_{abs} &= E'(s_{abs}) = mB\nu\beta(\kappa/\kappa_{gx})^{1/\alpha}
\end{aligned}$$

We assumed $\kappa_{gx} > 1$, $\nu > 0$ [$\beta > 0$; cf. (4.3)], so that E_{em}, E_{abs}, and E'_{abs} are always positive, but E'_{em} need not be. E'_{em} can become negative also for signals with $|\alpha| < 1$ and/or $\varepsilon = 1$. In fact, $E'_{em} > 0$ only if $\beta > (\kappa_{gx}^2 - 1)(\kappa_{gx}^2 + 1)^{-1}$. This criterion is essentially unrelated to the conditions for $t'_{abs} > t'_{em}$, cf. the discussion (I)–(III) after (5.12). Therefore a time inversion is not really connected with the appearance of negative energies as happens with tachyons in Minkowski space (Bilaniuk *et al.*, 1962; Feinberg, 1967). For example, if $\alpha = 1/\beta$ [i. e., $H = B$ in (2.6)], the sign change of energy [as defined in (4.18) with $\kappa \to \kappa/\kappa_{gx}$] occurs at the splitting point (5.9) of the trajectory. We then have $s'_E = s'_\infty$ with $s'_E := s_E(\kappa \to \kappa/\kappa_{gx})$; cf. (4.18). Thus P_1 has positive and P_2 negative energy; see the discussion of (5.13) (ε is necessarily -1 in this case, in order that $\alpha > 1$). As shown in the preceding discussion [(I)–(III) after (5.12)], whether t'_{abs} is larger or smaller than t'_{em} entirely depends on α.

Since the observations of O_1 and O_2 in their respective rest frames (t, x) and (t', x') may qualitatively differ, both observers have to reassemble their observations in comoving coordinates; cf. Sections 1 and 2. In this way observer O_2 can find out that the splitting of the trajectory and the possible time inversion $t'_{abs} < t'_{em}$ is an artifact of the geodesic coordinate frame which he arbitrarily chose as his rest frame. In comoving coordinates (τ, u) we have for the respective events (t_{em}, x_{em}) (t_{abs}, x_{abs}), (t'_{em}, x'_{em}), and (t'_{abs}, x'_{abs}):

$$\begin{aligned} \tau_{em} &= \kappa^{-1/\alpha}, \qquad u_{em} = 1 \\ \tau_{abs} &= (\kappa_{gx}/\kappa)^{1/\alpha}, \qquad u_{abs} = \kappa_{gx} \\ \tau'_{em} &= \tau_{em}, \qquad u'_{em} = \kappa_{gx}^{-1} \\ \tau'_{abs} &= \tau_{abs}, \qquad u'_{abs} = 1 \end{aligned} \tag{5.16}$$

We used here the isometry (2.2); the Lorentz boost (4.7) with $\mathbf{v} = \mathbf{v}_{gx}$ [cf. (5.2)] corresponds to a simple rescaling, $u' = \kappa_{gx}^{-1} u$, of the half-space coordinate. [As pointed out after (2.4) and (3.4), Lorentz transformations in the forward light cone correspond to symmetry transformations on the 3-slices $\tau = const.$]

We have $\tau_{abs} - \tau_{em} = \tau'_{abs} - \tau'_{em} = \kappa^{-1/\alpha}(\kappa_{gx}^{1/\alpha} - 1) > 0$, since either $\kappa_{gx} > 1$ and $\alpha > 0$, or $\kappa_{gx} < 1$ and $\alpha < 0$, as indicated after (5.12). In the comoving frame the signal moves straight from observer O_1 to O_2; there is no splitting of the trajectory nor a time inversion in sight in this universal frame of rest; cf. Section 1.

6. CONCLUSION

The cosmology developed here is based on a flat space-time continuum and a permeability tensor representing the substance of physical space. We focus in this paper on classical mechanics in the ether. The formalism is designed after geometric optics in a dielectric medium; the main difference is that the ether affects not only rays, but also massive particles. The scale factors in the isotropic permeability tensor are chosen so that the semiclassical approximation is exact. Therefore we can study the effects of the ether on the red-shift completely in the framework of eikonal and Hamilton–Jacobi equation. The energy concept in the ether is defined in such a way that the Einstein/de Broglie relation holds.

In RW cosmology there exists a universal frame of reference in which the galaxies have constant space coordinates. The space expansion determines a unique cosmic time. This comoving coordinate frame provides a common rest frame for all galactic observers. For each of these observers one can also introduce an individual, locally geodesic rest frame in which the galaxies radially recede. In the cosmology studied here individual rest frames are even globally geodesic and not restricted to infinitesimal neighborhoods. In geodesic coordinates red-shifts are Doppler shifts like in a dielectric medium.

In geodesic rest frames the speed of light is, due to the permeability of the ether, not constant, and depends both on space and time coordinates; cf. (3.8). At a given space-time point a massive particle may exceed the speed of light. There is no upper bound on the speed of signal transfer. Therefore it is natural to consider in this context particles (tachyons) with negative mass square [i.e., $\varepsilon = -1$ in (2.7)]. The speed of a tachyon may be smaller than the speed of light. Globally geodesic rest frames should not be mixed up with the local Minkowskian limit in the comoving frame, as discussed after (2.18). In this limit the speed of light is a universal constant, the mass depends on the cosmic time parameter, and energy is a positive-definite, conserved interaction parameter.

In geodesic rest frames it may happen that the world-line of a particle splits, and a double image of the particle appears. If in comoving coordinates the particle moves from space point A to B, it may happen in a geodesic rest frame that simultaneously two particles emerge at a point C, one moving to A and the other to B with opposite velocities. Even if there is no splitting of the trajectory, the signal may appear time-inverted in geodesic frames, namely moving from B to A. These effects are, however, just artifacts of individual rest frames. A galactic observer can always replace his geodesic coordinates by comoving ones, and so come to definitive conclusions on the cosmic time order of events, which is the same for all galactic observers.

The introduction of privileged coordinate frames in the context of general covariance was also suggested by Fock (1959) in terms of harmonic coordinate systems (defined by a trace condition on the connection coefficients). In cosmology the galactic background provides a universal rest frame for all galactic observers. One thus arrives at an absolute cosmic space-time, and on that basis it is straightforward to regard physical space as a permeable medium, the cosmic ether.

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